Alternating power supply

(L C R combinations and applications)

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$a.c.$ circuits

- Phasor diagram
- R, C and L combinations
- Resonance and quality factor
- Power and Power factor
- L-C oscillations
- Transformer

Phasor diagrams

- Phasor diagram is a convenient representation of current and the potential drops across the three elements.

  Let I be the phasor representing the current in the circuit. Let V_L , V_R , V_C , and V represent the voltage across the inductor, resistor, capacitor and the source, respectively.

- Length of each phasor is given by $I_{max}R$, $I_{max}X_C$ and $I_{max}X_L$.

- In a series combination of L, C and R connected to an a.c. power supply, current through each element is the same at any instant of time (both in magnitude and in phase) .

- Sum of potential drops across the elements (added “as vectors” including their phase angles) is equal to the potential of the source.

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L and *R* in series

$$V = V_o \sin(\omega t)$$ — i

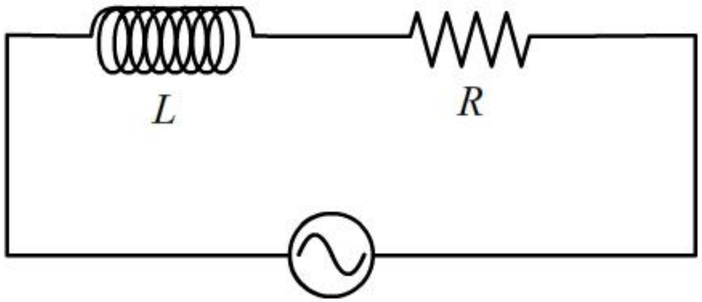
$$i = i_o \sin(\omega t - \phi)$$ — ii

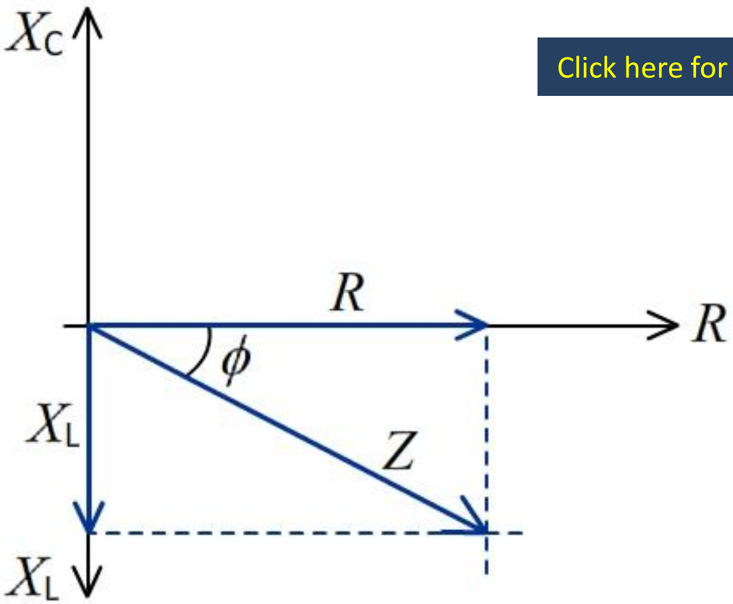
$$i_o = \frac{V_o}{Z}$$

$$Z = \sqrt{R^2 + X_L^{\,2}}$$ — iii

$$i_o = \frac{V_o}{\sqrt{R^2 + X_L^{\,2}}}$$ — iv

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$ — v





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a.c. circuits

C and *R* in series

$$V = V_o \sin(\omega t) \quad \text{i}$$

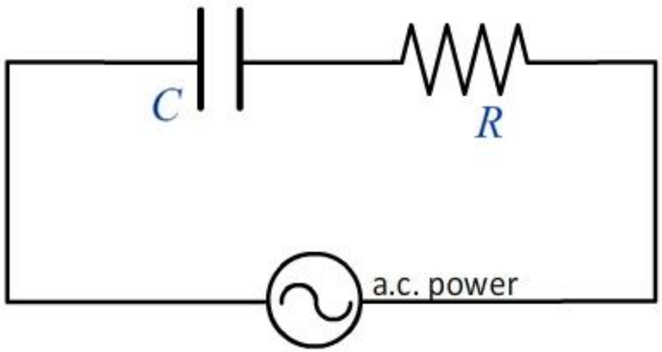
$$i = i_o \sin(\omega t + \phi) \quad \text{ii}$$

$$i_o = \frac{V_o}{Z}$$

$$Z = \sqrt{R^2 + X_C^{\ 2}} \quad \text{iii}$$

$$i_o = \frac{V_o}{\sqrt{R^2 + X_C^{\ 2}}} \quad \text{iv}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) \quad \text{v}$$



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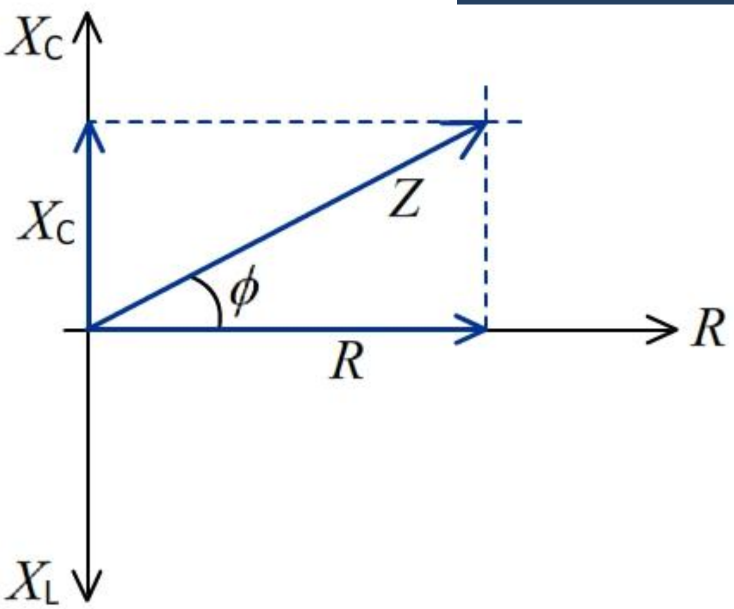


L, *C* and *R* in series

$$V = V_o \sin(\omega t) \quad \text{i}$$

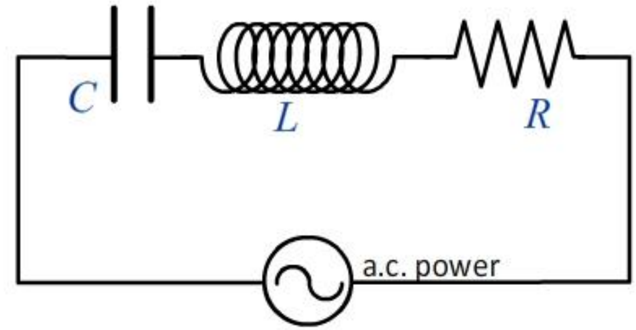
$$i = i_o \sin(\omega t + \phi) \quad \text{ii}$$

$$i_o = \frac{V_o}{Z}$$

Assuming $X_C > X_L$ we get

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \quad \text{iii}$$

$$i_o = \frac{V_o}{\sqrt{R^2 + (X_C - X_L)^2}} \quad \text{iv}$$



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As $X_C > X_L$ resultant reactance is *in the direction* of X_C. Therefore ϕ is positive and given by

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right) \quad \text{v}$$

If $X_C < X_L$ then the term (X_C - X_L) may be replaced by (X_L - X_C).
In such case ϕ is negative.

Power and power factor

In a.c. circuits instantaneous values of current and voltage are given by

$$V = V_o \sin(\omega t) \qquad i = i_o \sin(\omega t + \phi)$$

Substituting these in relation for power i.e. $P = VI$ we get

$$P = V_o \sin(\omega t)\, i_o \sin(\omega t + \phi)$$

$$P = V_o\, i_o \sin(\omega t)\left[\sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)\right]$$

$$P = V_o\, i_o \left[\sin(\omega t)^2 \cos(\phi) + \sin(\omega t)\cos(\omega t)\sin(\phi)\right]$$

Average power is given by

$$\boxed{\langle P \rangle = \frac{V_o\, i_o}{2}\cos(\phi)}$$

$$\boxed{\langle P \rangle = V_{rms}\, i_{rms} \cos(\phi)}$$

$\cos(\phi)$ is called the power factor of the *a.c.* circuit

Resonance in *a.c.* circuit with LCR

Resonance is a condition in a series combination of L, C and R connected to an a.c. power supply when inductive reactance (X_L) is equal to capacitive reactance (X_C).

At resonance impedance in the circuit becomes minimum and the power dissipated in the circuit is the maximum.

Resonant frequency

At resonance $X_L = X_C$ therefore

$$L\omega = \frac{1}{C\omega}$$

$$\Rightarrow \omega^2 = \frac{1}{LC}$$

$$\boxed{\omega_o = \sqrt{\frac{1}{LC}}}$$

ω_o : Resonant frequency

Impedance at resonance

Impedance in a general LCR circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance $X_L = X_C$ therefore

$$Z = \sqrt{R^2}$$

$$\boxed{Z_{\text{atresonance}} = R}$$

Impedance is minimum at resonance

Resonance in *a.c.* circuit with *LCR*

Phase angle (ϕ) at resonance

Phase angle is given by

$$\tan(\phi) = \frac{(X_L - X_C)}{R}$$

At resonance $X_L = X_C$ therefore

$$\tan(\phi) = \frac{0}{R}$$

$$\boxed{\phi_{\text{at resonance}} = 0}$$

Current is in phase with applied voltage

Power dissipated at resonance

Power dissipated in a general *LCR* circuit is given by

$$\langle P \rangle = V_{\text{rms}} I_{\text{rms}} \cos(\phi)$$

At resonance $\phi = 0$ therefore the power factor i.e. cos (ϕ) is 1

$$\langle P \rangle = V_{\text{rms}} I_{\text{rms}}$$

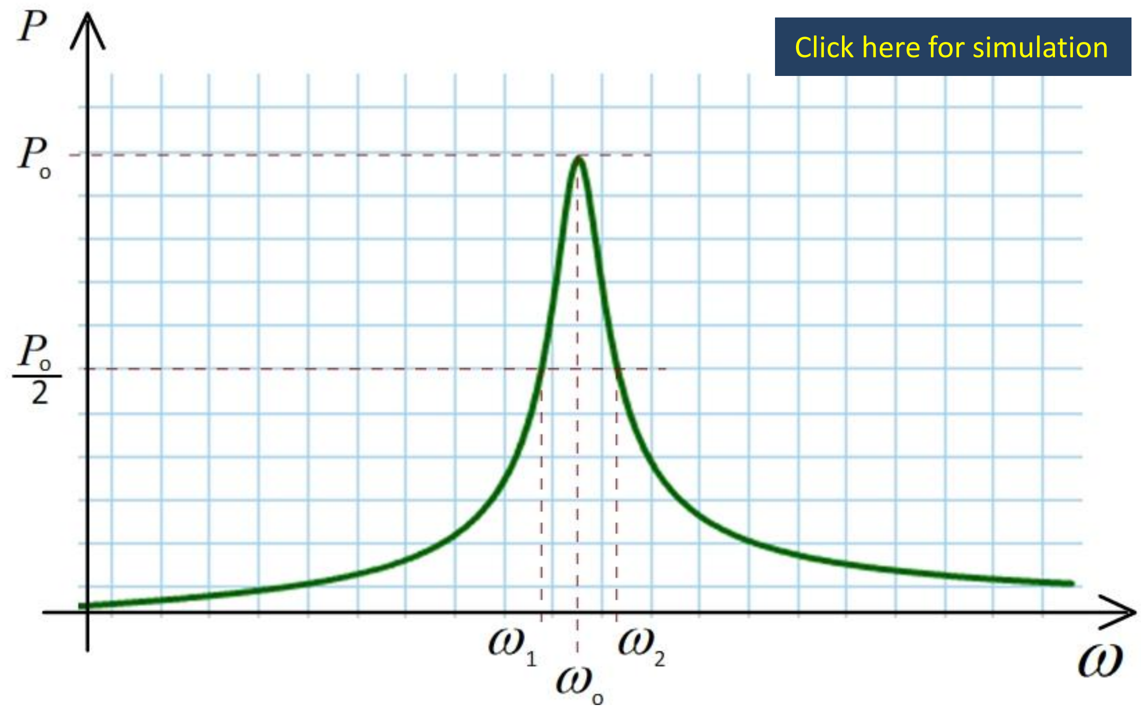
$$\boxed{\langle P \rangle_{\text{at resonance}} = V_{\text{rms}} I_{\text{rms}} = \frac{V_{\text{rms}}^{2}}{R} = I_{\text{rms}}^{2} R}$$

Power dissipated is the maximum at resonance

Quality factor

<u>Sharpness of resonance</u>

Sharpness of resonance is determined by quality factor (Q) which is given by



$$\boxed{Q = \frac{\omega_o}{2\Delta\omega}}$$

$$\boxed{Q = \frac{\omega_o L}{R}} \qquad \boxed{Q = \frac{1}{\omega_o CR}}$$

*Quality factor is also called selectivity.

ω_o : resonant frequency

ω_1 & ω_2 : two side frequencies at which the power dissipated in the circuit reduces to half

L : inductance of the inductor

R : resistance of the resistor

R : capacitance of the capacitor

Quality factor

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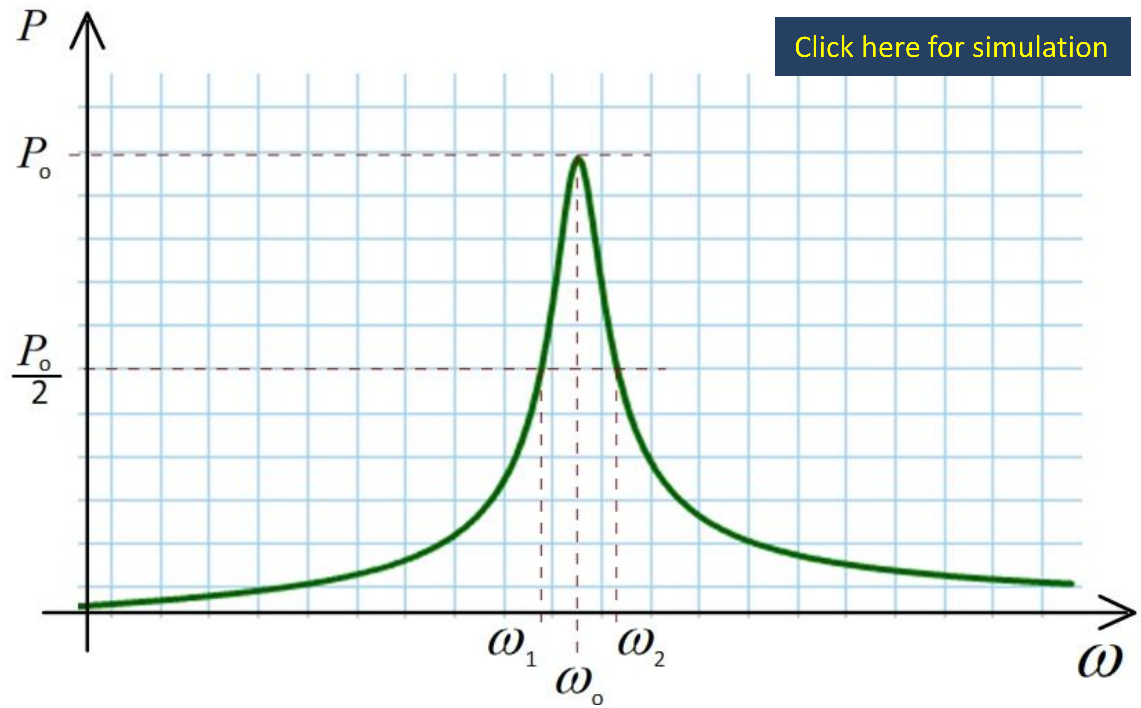
Bandwidth

$$\Delta\omega = \omega_2 - \omega_1$$

ω_o : resonant frequency

ω_1 & ω_2 : two side frequencies at which the power dissipated in the circuit reduces to half



When the resonance is less sharp , peak current decreases and a large range ($\Delta\omega$) of frequencies are close to resonant frequency. This results in less effective tuning.

$L - C$ oscillations

When a capacitor (initially charged) is connected to an inductor, the charge on the capacitor and the current in the circuit exhibit periodic oscillations similar to SHM.

Total energy in the system i.e. electric energy stored in the capacitor is distributed between capacitor as electric energy and inductor as magnetic energy.

Angular frequency of these electric oscillations is given by

$$\boxed{\omega_o = \sqrt{\frac{1}{LC}}}$$

Finite resistances of the connecting wires and the inductor, result in dissipation of energy.

Comparison of electric & mechanical phenomena

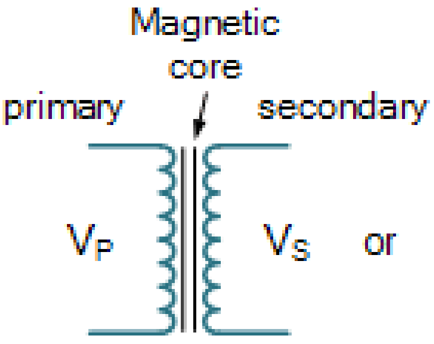
Mechanical oscillations	Electric oscillations
Mass (M)	Inductance (L)
Force constant (K)	$1/C$
Displacement (x)	Charge (q)
Velocity (v)	Current (i)
Kinetic energy $\frac{1}{2}mv^2$	Magnetic energy $\frac{1}{2}Li^2$
Potential energy $\frac{1}{2}Kx^2$	Electric energy $\frac{Q^2}{2C}$

Transformer

Transformer is a device used to increase/decrease the voltage of an *a.c.*

Transformer works on the principle of mutual induction.

A transformer consists of two sets of coils, insulated from each other. Both the coils are wound over a soft Ferro-magnetic core.



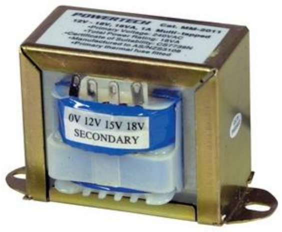
Coil connected to the input is called the primary coil and the coil from which the output is obtained is called secondary coil.

Number of turns in input and output coils are denoted as N_p and N_s respectively.

Step up transformer : Output voltage is greater than input voltage ($N_s > N_p$)

Step down transformer : Output voltage is lesser than input voltage ($N_s < N_p$)

$$\boxed{\mathcal{E}_{out} = \frac{N_s}{N_p}\mathcal{E}_{in}} \qquad \boxed{i_{out} = \frac{N_p}{N_s} i_{in}}$$



There is no gain or loss of power in an ideal transformer

Phase difference between the output and input voltage is 180°